Probability of Combined Events

Notes by : rraezzz

COMBINED EVENTS

♥ Are the combination of two or more events in an outcome

The possible number of outcomes is $3 \times 2 = 6$.

In general,

$$n(S) = n(A) \times n(B)$$

$n(S)$ is all the possible number of outcomes, $n(A)$ and $n(B)$ represent the number of outcomes of event A and event B.

Example 1

Write the sample spaces for the combined events below.

(a) Five cards labelled with the letters "T, E, K, U, N" are put into a box. Two cards are taken out at random from the box one by one without replacement.

(b) Two coins are tossed (T and H representing tails and heads respectively).

Solution:

(a) {(T, E), (T, K), (T, U), (T, N), (E, T), (E, K), (E, U), (E, N), (K, T), (K, E), (K, U), (K, N), (U, T), (U, E), (U, K), (U, N), (N, T), (N, E), (N, K), (N, U)}

(b) {(T, T), (T, H), (H, T), (H, H)}

DEPENDENT EVENTS AND INDEPENDENT EVENTS

Example 2

Identify whether the following combined events are dependent events or independent events. Justify your answers.

(a) Obtain a tail twice when a fair coin is tossed twice.

(b) Obtain a tail in tossing a fair coin and obtained the number 4 in rolling a fair dice.

(c) Obtain two pens of the same colour when two pens are taken out one by one from a container that contains three red pens and two blue pens without replacement.

(d) Obtain two cards with the same letters when two cards are chosen at random from the cards labelled with the letters "B, A, I, K" one by one with replacement.

Solution:

(a) Independent events because the probability of getting a tail in the first toss does not affect the probability of getting a tail in the second toss.

(b) Independent events because the probability of getting a tail in tossing a fair coin does not affect the probability of getting the number 4 in rolling a fair dice.

(c) Dependent events because the probability of getting the first red pen affects the probability of getting the second red pen.

(d) Independent events because the probability of choosing the first card does not affect the probability of choosing the second card.

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FORMULA OF PROBABILITY OF COMBINED EVENTS

The probability of the intersection of two independent events A and B is equal to the product of the probability of event A and the probability of event B.

In general,

Multiplication rule of probability is $P(A \text{ and } B) = P(A) \times P(B)$

Example 3

Box F contains seven cards labelled with the letters "P, A, M, E, R, A, N" and box G contains five cards labelled with the numbers "3, 5, 6, 8, 11". A card is chosen at random from box F and box G respectively. Verify the conjecture about the formula of probability to get the letter "P" and an even number by listing all the possible outcomes.

Solution:

(a) Multiplication rule

$P(\text{getting a letter "P"}) = \frac{1}{7}$

$P(\text{getting an even number}) = \frac{2}{5}$

$P(\text{getting a letter "P" and an even number}) = \frac{1}{7} \times \frac{2}{5}$

$= \frac{2}{35}$

Indicator

$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

List all the possible outcomes.

Possible outcomes = {(P, 6), (P, 8)}

$n(S) = 7 \times 5$

$= 35$

$P(\text{getting a letter "P" and an even number}) = \frac{2}{35}$

Therefore, it is shown that both methods give the same answer.

MUTUALLY EXCLUSIVE EVENTS AND NON-MUTUALLY EXCLUSIVE EVENTS

A and B are Mutually Exclusive Events if $A \cap B = \phi$,
$P(A \cup B) = P(A) + P(B)$

Example: $X = \{x : 1 \leqslant x \leqslant 10, x \in \mathbf{W}\}$
A number is chosen at random from set X.
Probability of choosing the number 2 or an odd number
$= P(\text{Number 2 or an odd number})$

$= \frac{1}{10} + \frac{5}{10}$

$= \frac{3}{5}$

A and B are Non-Mutually Exclusive Events if $A \cup B \neq \phi$,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: $X = \{x : 1 \leqslant x \leqslant 10, x \in \mathbf{W}\}$
A number is chosen at random from set X.
Probability of choosing a prime number or an odd number
$= P(\text{A prime number or an odd number})$

$= \frac{4}{10} + \frac{5}{10} - \frac{3}{10}$

$= \frac{3}{5}$